



Article

Coordinating Promotion for Household Energy-Conservation

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Abstract

This paper develops a game-theoretic model to analyze the coordination between an energy distribution company (EDC) and a loyalty reward program (LRP) marketing firm for promoting household energy conservation. The partnership is characterized by a break-even LRP operator and a profit-driven EDC subject to a regulatory conservation target. We examine both non-cooperative Stackelberg games and a cooperative game. Our analysis yields several key insights. First, we find that a non-cooperative structure with the EDC as the leader can achieve system-wide profit equal to that of a fully cooperative scenario, providing a practical, implementable coordination mechanism. Second, the ratio of the perceived value to the redemption cost of reward points emerges as a pivotal contingency factor that determines the effectiveness of advertising spending and energy pricing strategies. Furthermore, reward points cannot fully overcome high household switching costs, and average energy savings show no significant impact on participation rates. These findings offer actionable guidance for designing efficient public-private partnerships in energy demand-side management and contribute to the literature on applying market-based incentives to pro-social behavioral change.

Keywords: loyalty reward program; household energy conservation; public-private partnership; game theory; coordination

1. Introduction

The energy sector's key task is to balance energy supply and demand while achieving sustainable development. Due to the increasing environmental concerns, household energy-conservation is recommended as one of the most inexpensive and feasible ways to reduce



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energy demand and meet climate-change mitigation targets (Breuker et al. 2011). Numerous studies have addressed environmental, social, psychological, and economic issues in encouraging household energy-conservation and managing residential energy demand (e.g., Jamasb and Pollitt 2011). However, altering energy-conservation behavior remains the most challenging and sophisticated issue in energy demand management (Steg and Vlek 2009). In this research, we examine an incentive-based marketing program known as a reward program or loyalty program (simplified as LRP hereafter) and its application for encouraging household energy-conservation behaviors.

LRP is a type of marketing program that “encourages repeat buying and thereby improves retention rates by providing incentives for customers to purchase more frequently and in larger volumes”. It is also known as frequent flyer/shopper program or club/membership program, adopted in a wide range of consumer goods and service industries, particularly in airline, hotel, financial service, and retail industries to enhance customer relationships and long-term profitability (e.g., Khirallah, 2005, Frequent-flyer miles 2005, Nunes and Drèze 2006, Rowley 2007). Both practical applications and theoretical research show that well-designed and well-operated LRPs can effectively alter consumer decisions and behaviors (e.g., Liu 2007, Henderson 2011). In recent years, due to the growth of LRP-related “loyalty” markets and the increasing economic and social impact of LRPs, practical applications of these programs have emerged in non-traditional areas such as healthcare services, and environmental advertising and communications. Recent studies in these areas show that LRPs are capable of using both financial and psychological incentives to increase public awareness and information, provide tailored feedback, and target heterogeneous consumer segments (e.g., OPA 2011).

This paper was developed from a research project designed for a power agency and its LRP partner in Canada. The power agency recently partnered with an LRP enterprise and initiated a pilot application of an LRP to promote a province-wide, high-profile energy-conservation program designed to motivate households to take simple conservation actions that have lasting financial benefits and help fight climate change (LoyaltyOne 2011). The power agency and its LRP partner are seeking academic advice on how to address operational issues in this partnership and how to improve future energy-conservation programs. In this paper, we consider an LRP application in which a local energy distribution company (EDC) partners with an LRP marketing firm to jointly promote household energy-conservation programs. In this LRP application, EDC is a profit-driven business entity, whereas the LRP marketing firm operates on a cost-recovery (break-even) basis, meaning it sets prices to cover its operational costs without seeking profit from this partnership. The LRP marketing firm will sell incentives (e.g., reward miles or reward points) at a given price to the EDC, and the EDC passes on those incentives to households who participate in the energy-conservation program. Although the EDC is a profit-driven business entity, it must take some social responsibility. A general practice

in the energy industry is that EDCs are required to meet some energy-conservation targets. EDCs face a penalty if they cannot meet that target; whereas if they do, they receive a credit.

While the marketing literature offers a rich repository of theories and models on LRPs and consumer incentives, our study distinguishes itself by transplanting these insights into the distinct context of public-policy-driven household energy conservation. Unlike conventional commercial settings, our model incorporates the unique objectives of a regulated energy distributor (balancing profit with mandated conservation targets) and a break-even LRP operator. This shift in scenario fundamentally alters the strategic interplay, moving the research focus from maximizing customer equity to optimizing the procurement of societal behavioral change under a public-private partnership framework. Consequently, our analysis introduces and examines variables such as government-imposed conservation targets, penalty/credit mechanisms, and household switching costs in energy use—factors largely absent in mainstream LRP studies. The primary contribution of this paper, therefore, lies not in revisiting established LRP theories, but in developing and solving a novel model that addresses the operational coordination challenges inherent in using market-based loyalty incentives to achieve public energy conservation goals, thereby generating unique managerial and policy insights.

To the best of our knowledge, neither the application of LRP to the energy sector nor the collaboration between LRP marketing firms and energy agencies has been studied in the literature. In this paper, we develop analytical models to examine key operational decisions involved in this LRP application, particularly the decisions on optimal level and pricing of LRP reward points. We also examine the impact of the key characteristics of the LRP and energy-conservation program on household participation, as well as on the profit and the cost allocations between the EDC and the LRP marketing firm. To capture the decision processes and potential gaming effect between the EDC and the LRP marketing firm, we implement two-stage games that describe the interactions between the two firms and analyze the equilibrium solutions of the games. We consider two decision scenarios in the games: (1) The EDC is the leader, and the LRP marketing firm the follower (Case1); (2) The LRP marketing firm makes its decision first, and the EDC follows (Case2). We then examine the conditions under which a specific equilibrium might happen in those scenarios. Our analysis shows that the household participation rate at the equilibrium point in Case1 is always no less than that in Case2. Meanwhile, in Case1, the LRP marketing firm and EDC can achieve higher system-wide profit than they do in Case2. We find that when the LRP marketing firm manages its revenue and profit by break-even, if the EDC acts as the leader in the non-cooperative game, then the non-cooperative game generates the same amount of system-wide profit as in the cooperative game. The EDC obtains more profit and spends less money to purchase LRP incentives (i.e., reward points). Meanwhile, more households will join/participate in the energy-conservation program.

We also find that the advantages of using LRP are more obvious when the LRP incentive (i.e., reward points) is more attractive to households, and/or the redemption cost of the LRP incentive is reduced to a certain level (e.g., the EDC can reduce their advertising expenses without lowering household participation rate and system-wide profit.) Households are less sensitive to the economic benefits, in terms of average energy-price reduction, obtained from joining/participating in an energy-conservation program.

In addressing this practical coordination problem, this study aims to make several theoretical contributions. First, it seeks to extend marketing theories on loyalty programs by examining their adaptation as tools for 'social behavior procurement' within a regulated, non-competitive context—a significant shift from their traditional commercial objectives. Second, it aims to advance the energy policy literature by developing a micro-founded, strategic model of a market-based incentive scheme, complementing existing macro-level evaluations. Finally, it contributes to operations management theory by analyzing coordination mechanisms between partners with fundamentally asymmetric objectives (profit vs. break-even) under an external policy constraint. By bridging these literatures, this paper provides a novel analytical framework for understanding and optimizing public-private partnerships aimed at behavioral change.

2. Literature Review

Our paper is related to three streams of research: energy demand management, LRP, and supply chain contracting.

2.1. Energy Demand Management

The first stream is about energy demand management. Researchers in various disciplines have extensively studied energy consumption and conservation and its related problems. Breukers et al. (2011) provide a comprehensive overview of energy demand-side management studies in multiple disciplines. In the economic literature, many studies focus on energy-demand modeling at the macro-level and rely on advanced econometric methods for demand analysis. Suganthi and Samuel (2012) provide a comprehensive review of those approaches and classify them into twelve categories. Gouda and Aly (2025) aims to develop an energy management strategy for distribution grids (DGs) by incorporating a hydrogen storage system (HSS) and demand-side management strategy (DSM). Tsai and Tsai (2025) employs the Grey-Markov Model (GMM) to predict users' electricity demand and introduces the Enhanced Monte Carlo (EMC) method to assess the reliability of the prediction results. In the studies on energy efficiency and conservation, many focus on how to correct market failure and remove barriers to adopting and improving energy efficiency (e.g., Filippini and Hunt, 2012, Maslak et al. 2025, Tan et al. 2025, Habib et al. 2026, Nkala et al. 2025, Kanetaki et al. 2025, Sanglay et al. 2025,

Nalabolu et al. 2025). Other researchers examine consumer preferences and decision-making in energy efficiency and conservation (Tamilarasi and Gandhimohan 2025).

Many studies have pointed out that consumers often undervalue the benefits of energy efficiency and conservation; therefore, interventions should be undertaken. Various interventions aimed at encouraging household pro-environmental behavior have been studied and evaluated, including financial or psychological incentives such as loads, rebates, feedback (e.g., Hargeaves et al. 2010). The practice of using financial incentives (e.g., Deichmann and Zhang 2013) or psychological incentives to encourage energy-conservation or efficiency is highlighted in several studies (Allcott and Greenstone 2012).

Financial/material incentives have proven effective in altering energy-saving decisions. We examine household decision-making processes in energy consumption from a behavioral economics perspective. As Bhattacharyya & Timilsina (2009) pointed out, any energy use involves costs and resource-allocation problems. The household decision-making process for energy use includes three types of decisions: switching, selection, and consumption. Our paper focuses on the household decision-making process of whether to switch to an energy-conservation program or not. In order to capture households' heterogeneous preferences, we use a vertical product differentiation model (Kunin and Zigic 2022). In that model, we integrate the consideration of LRP incentives, the advertising expenditures of both the LRP marketing firm and the EDC, and their impact on household decision making in the context of energy-conservation.

2.2. loyalty Programs (LRPs)

The second stream of related literature is studies on LRP. A number of marketing studies have investigated the impact, design, and effectiveness of LRP. Those studies can be classified into the following three main research streams: (1) design and implementation of LRP (e.g., You et al. 2025, Dionglay and Callo. 2024, Ong et al. 2024, Farthing et al. 2025, Binder et al. 2025, Sharma 2025, Zhang et al. 2025, Fan et al. 2026, McDermott et al. 2025); (2) short- or long-term impact of LRP on consumer purchase behavior, attitudes, and decisions (e.g., Liu 2007, Meyer-Waarden 2007, 2008, Gandomi and Zolfaghari 2011, Yoo 2011); and (3) influence of LRP on a firm's decisions and market competition (e.g., Kim et al. 2004, Kopalle and Neslin 2003, Singh et al., 2008, Liu and Yang 2009).

Our research contributes to the marketing and LRP literature not by re-examining the traditional firm-customer dyad, but by extending the investigation to a triadic structure involving a public-policy-influenced energy firm, a non-profit-seeking LRP operator, and households. We explore how LRP, as an incentive mechanism, functions within this constrained, goal-oriented partnership to influence pro-environmental consumer decisions—a context underexplored in existing LRP studies. This shift allows us to examine novel research questions, such as the optimal coordination mechanism when one partner operates at break-even and the system faces an external conservation target, yielding equilibrium results (e.g., the

equivalence of profits in a specific leader-follower game and the cooperative game) that are distinctive to this hybrid operational setting.

2.3. Supply Chain Contracting

Finally, literature on supply chain contracting is vast (Katok and Wu 2009, Selviaridis and Spring 2018, Li et al. 2021, Mfonnom 2025, Gong et al. 2026, Brugués 2026, Dong et al. 2024., Ellati et al. 2025, Papier et al. 2025). Bolton and Dewatripont (2005), and He and Zhao (2012) provide excellent reviews. Coordination issues for a sustainable energy supply chain that involves public and private partnerships have not been studied. Our research will contribute to this literature by investigating the coordination and partnership between LRP marketing firms and energy service providers for altering energy demand and meeting energy-conservation targets.

Theoretically, this study contributes to three distinct but interconnected literatures. First, to the marketing literature on loyalty/reward programs, it extends the theoretical boundary conditions by modeling an LRP not as a tool for customer retention or profit maximization in a competitive market, but as a 'social behavior procurement' mechanism within a regulated, non-competitive environment. This shift challenges core assumptions about firm objectives and redefines the key metrics of program success. Second, it contributes to the energy economics and policy literature on demand-side management by providing a micro-foundational, game-theoretic model of a market-based incentive scheme. It moves beyond macro-level policy evaluation or pure engineering models to analyze the strategic interaction between the implementing agents. Finally, it informs the operations and supply chain management literature on coordination by modeling a dyad with profoundly asymmetric objective functions (profit-maximizer vs. break-even operator) under a binding, external constraint—a configuration common in public-private partnerships but underexplored in coordination models.

The rest of this paper is organized as follows: the model is formulated in Section 3. Section 4 describes the procedure to solve the model and find equilibrium solutions. Section 5 discusses the key findings and managerial insights. In Section 6, a conclusion with a summary on model extension and future research is provided.

3. Model Development

3.1. Key Characteristics of the Residential Energy Sector and Modeling Implications

It is important to highlight several defining characteristics of the residential energy sector that shape the context and assumptions of our model, distinguishing it from conventional consumer markets. First, retail energy distribution is typically a regulated monopoly or oligopoly, meaning households are captive customers of a single, geographically defined utility. Consequently, the Energy Distribution Company (EDC) in our model is not using the LRP to compete for market share but to manage the demand of its existing customer base to meet external policy objectives. Second, a core driver for the EDC is regulatory compliance; it

operates under mandated energy conservation targets (T_E) with associated financial penalties or credits (c_2), making conservation a primary operational goal rather than a voluntary initiative. Third, household energy consumption is often a low-engagement, habitual behavior. The direct economic benefit for a household from conservation—a reduced energy bill—is typically small, delayed, and thus a weak motivator on its own (β). This justifies the need for a more salient, immediate incentive like LRP points to alter behavior and explains the significant role of advertising (A, B) in overcoming consumer inertia. Finally, this context creates the necessity for the public-private partnership we study: the EDC possesses the customer relationship and regulatory mandate but often lacks specialized marketing expertise, while the LRP firm provides the engagement platform and operational capability, frequently on a cost-recovery (break-even) basis. These sector-specific features directly inform our modeling choices regarding objectives, constraints, and household decision-making.

3.2. Model Formulation and Assumptions

We consider an LRP application in which a local energy distribution company (EDC) as a for-profit energy service provider partners with an LRP marketing firm to jointly promote household energy-conservation activities. In this cooperative promotion for a household energy-conservation initiative, the LRP marketing firm manages its costs and revenue by break-even, which is the case implemented in the real-world application as described in section 1). To implement this LRP application, the LRP marketing firm needs to decide w , the selling price of the LRP incentive, and the EDC needs to decide e , the amount of the incentive given to each participating household. The LRP incentive here refers to reward points, the most popular form of virtual incentives issued by LRP marketing firms. For the LRP marketing firm, the main costs incurred in the operation of this LRP application are the redemption cost of LRP incentives and its advertising spending. The redemption cost of LRP incentives refers to the money that LRP marketing firm spends on rewarding households. For the EDC, the main costs are the potential penalty (or credit) associated with meeting (or missing) the regulatory conservation target, as well as the EDC's advertising expense.

In our study, we consider an energy market that consists of multiple household segments i , $i=1, \dots, N$. The proportion of each household segment in the energy market is known as λ_i , and

$$\sum_{i=1}^N \lambda_i = 1.$$

In this paper, we apply a vertical product differentiation model to describe the household decision-making process of whether to join an energy-conservation program. Each household's

preference on whether to participate in the energy-conservation program or not can be modeled as each household's purchase decisions between two differentiable products. In the context of energy-conservation, we assume there are two products in the residential energy market: product R and product E . Product R refers to the regular energy program, which represents households' regular energy consumption status. Product E refers to the energy-conservation program that EDC promotes. The cost that a household in segment i incurs for switching from product R (the regular energy program) to product E (the energy-conservation program) is denoted by v_i . This term represents the household switching cost, which captures the overall investment (e.g., time, money, and effort) required to participate in the conservation program, beyond the direct financial outlay. Consequently, we set the switching cost for staying with product R , $v_{R(i)} = 0$, while $v_{E(i)} = v_i > 0$ for choosing to switch to product E . The household's decision is also influenced by the average energy price (e.g., combining peak and off-peak energy consumptions and prices) the non-participating household pays, p_R , and the participating household pays, p_E . A reasonable assumption here is that $p_R \geq p_E$. Moreover, a household's decision will also be influenced by the number of reward points, e that the household expects to receive, and the advertising expenditures A and B that the EDC and LRP marketing firm spend respectively on the energy-conservation program. We assume that the impacts of e , A , and B on households' decision to participate are the same across household segments.

We assume a household in segment i has a preference parameter θ_i uniformly distributed between $[\underline{\theta}_i, \bar{\theta}_i]$. If the household joins in the energy-conservation program with energy-conservation level $s_{E(i)}$, the utility they get is $\theta_i s_{E(i)}$ for each unit of consumption, then the household's surplus function for each unit of energy consumption is:

$$U_{E(i)} = \theta_i s_{E(i)} - v_{E(i)} - p_E + g(e), \quad i = 1, \dots, N \quad (1)$$

Here, $g(e)$ represents the household benefit obtained from incentives (e.g., reward points) offered by the LRP marketing firm for each unit of consumption under the conservation program. In real-world applications, LRP incentives normally have perceived value (e.g., 95

reward miles = \$10 off in store¹) to consumers. To consumers who collect these reward points/miles, more points/miles means more redeemable value to exchange for future rewards. Therefore, we assume $g(e)$ increases linearly with respect to e . Specifically, we assume $g(e) = \beta e$, where β is a positive constant, representing the benefit per unit of points that a household receives.

If the household does not want to participate in the energy-conservation program, and the household energy-conservation level in the regular program is denoted by $s_{R(i)}$, then the utility they get is $\theta_i s_{E(i)}$ for each unit of consumption, and the household's surplus function for each unit of energy consumption is:

$$U_{R(i)} = \theta_i s_{R(i)} - v_{R(i)} - p_R, \quad i = 1, \dots, N \tag{2}$$

A reasonable assumption here is that product E has higher energy-conservation than product R , $s_{R(i)} \leq s_{E(i)}$. A household in segment i with the preference parameter θ_i is indifferent between the two products if and only if $U_{E(i)} = U_{R(i)}$. In other words, to the household, there is no difference between the two options (switch to the energy-conservation program or stay with the regular energy program). Let $\tilde{\theta}_i$ denotes the value of θ_i when $U_{E(i)} = U_{R(i)}$, then

$$\tilde{\theta}_i = \frac{v_{E(i)} - v_{R(i)} + p_E - p_R - g(e)}{s_{E(i)} - s_{R(i)}} = \frac{v_i + p_E - p_R - \beta e}{\Delta s_i}, \quad i = 1, \dots, N, \tag{3}$$

where $v_{R(i)} = 0$, $v_{E(i)} = v_i > 0$, and $\Delta s_i = s_{E(i)} - s_{R(i)} \geq 0$ for all i .

Based on the above result, we have: households choose product E (i.e., join the energy-conservation program) when $\theta_i \geq \tilde{\theta}_i$, and households choose product R (i.e., stay with the regular energy program) when $0 \leq \theta_i \leq \tilde{\theta}_i$. We assume that the household preference with

¹written on Air Miles website: https://www.airmiles.ca/arrow/Overview?_requestid=475690 (accessed on June 2nd, 2014).

respect to the energy-conservation program θ_i follows uniform distributions with a lower bound $\underline{\theta}_i$ and an upper bound $\bar{\theta}_i$. The density function of θ_i is as follows:

$$F(\tilde{\theta}_i) = \frac{v_i + p_E - p_R - \beta e - \underline{\theta}_i \times \Delta s_i}{\Delta \theta_i \times \Delta s_i}, \text{ where } \Delta \theta_i = \bar{\theta}_i - \underline{\theta}_i \geq 0 \text{ for all } i \quad (4)$$

$F(\tilde{\theta}_i)$ represents the probability that each household will refuse to join in energy-conservation program based on its surplus. However, due to the advertising efforts of the LRP marketing firm and the EDC, this probability will be reduced to:

$$F_i = F(\tilde{\theta}_i) \times (1 - k_A \sqrt{A} - k_B \sqrt{B}), \text{ where } 0 < k_A \sqrt{A} + k_B \sqrt{B} < 1 \quad (5)$$

In Eq.(5), A denotes the LRP marketing firm's advertising spending, and B denotes the EDC's advertising spending. k_A and k_B represent the efficacy of each advertising effort in household demand for the energy-conservation program, $k_A > 0$, and $k_B > 0$. $k_A \sqrt{A}$ and $k_B \sqrt{B}$ capture common properties of advertising effects discussed in the literature (e.g., Karray and Zaccour 2006, Xie and Wei 2009): (1) demand response is an increasing and concave function with respective to advertising spending; (2) additional advertising spending generates continuously diminishing returns. In Eq.(5), reducing the probability of refusing to participate in the energy-conservation program is equivalent to increasing the probability of participation $1 - F_i$.

Let D denotes the size of energy market (i.e., the total number of targeted households in the energy market); λ_i denotes the proportion of each household segment i in the energy market,

and $\sum_{i=1}^N \lambda_i = 1$, where N is the total number of household segments.

The household demand for the energy-conservation program is as follows:

$$\begin{aligned}
 D_E &= D \times \sum_{i=1}^N [\lambda_i \times (1 - F_i)] \tag{6} \\
 &= D \times \sum_{i=1}^N \left[\lambda_i \times \left(1 - F(\tilde{\theta}_i) \times \left(1 - k_A \sqrt{A} - k_B \sqrt{B} \right) \right) \right] \\
 &= D \times \left[1 - f_{AB} \times \sum_{i=1}^N \lambda_i \times \left(\frac{v_i + p_E - p_R - \theta_i \times \Delta s_i}{\Delta \theta_i \times \Delta s_i} - \frac{\beta \times e}{\Delta \theta_i \times \Delta s_i} \right) \right] \\
 &= D \times (1 - f_{AB} \times k_0 + f_{AB} \times k_1 \times \beta \times e)
 \end{aligned}$$

where $f_{AB} = 1 - k_A \sqrt{A} - k_B \sqrt{B}$, $k_0 = \sum_{i=1}^N \lambda_i \times \frac{v_i + p_E - p_R - \theta_i \times \Delta s_i}{\Delta \theta_i \times \Delta s_i}$, $k_1 = \sum_{i=1}^N \frac{\lambda_i}{\Delta \theta_i \times \Delta s_i}$.

Let $m_0 = 1 - f_{AB} \times k_0$ and $m_1 = f_{AB} \times k_1$, Eq.(6) is rewritten as Eq. (7).

$$D_E = m_0 + m_1 \beta e, \tag{7}$$

Note in Eq. (7) the maximum value for D is normalized to be 1 for simplicity of the expression. A reasonable assumption here is that $m_0 \geq 0$, since when $e = 0$, $D_E = m_0 \geq 0$.

The profit of the LRP marketing firm is as follows:

$$\pi_{LRP} = w \times e \times D_E - (1 - \gamma) \times c_e \times e \times D_E - A \tag{8}$$

where γ denotes the breakage factor, which represents the percentage of reward points collected but that will never be redeemed by consumers (e.g., households). c_e is the per-unit redemption cost of reward points. In Eq. (8), π_{LRP} refers to the profit that the LRP marketing firm obtains from offering incentives to promote the energy-conservation program. The first term in the right-hand side of the equation is the revenue that the LRP marketing firm obtains by selling reward points to the EDC. The second term is the LRP marketing firm’s variable cost incurred in this LRP application, which is the redemption cost of reward points, and the third term is the LRP marketing firm’s fixed cost, which is the advertising expense for promoting the energy-conservation program.

The profit of EDC is as follows:

$$\pi_{EDC} = p_R \times q_R \times (1 - D_E) + p_E \times q_E \times D_E - B - w \times e \times D_E - c_2 \times (T_E - D_E) \tag{9}$$

In Eq. (9), π_{EDC} refers to the overall profit that the EDC obtains from distributing energy to households and collaborating with the LRP marketing firm to promote the energy-conservation program. We denote in Eq. (9) the energy-conservation promotion target by T_E , and the penalty (or credit) if the EDC cannot (or can) meet the target by c_2 , the average energy consumption of a typical household that participates in the energy-conservation program by q_E , and the average energy consumption of a typical non-participating household by q_R . On the right-hand side of Eq. (9), the sum of the first term and the second term is the revenue that the EDC obtains from distributing energy to households. The third term represents the EDC's advertising costs. The fourth term is the amount that the EDC spends to purchase reward points from the LRP marketing firm. In addition, the EDC must meet an energy-conservation target. The fourth term is the penalty that the EDC must pay if they do not meet the energy-conservation target or the credits that the EDC obtains if it does reach the target. The EDC's key decision variable is e , the number of reward points offered to each participating household. This determines the strength of the immediate incentive for households.

Combining (8) and (9), the entire system profit is as follows:

$$\begin{aligned} \pi_{LRP+EDC} &= \pi_{LRP} + \pi_{EDC} \\ &= p_R \times q_R \times (1 - D_E) + p_E \times q_E \times D_E - B - (1 - \gamma) \times c_e \times e \times D_E - A - c_2 \times (T_E - D_E) \end{aligned} \tag{10}$$

4. Model Solution

4.1. Distinctive Features of the Strategic Interaction

While the Stackelberg game structure employed is standard in the literature, the strategic interaction modeled here possesses several distinctive features that collectively differentiate it from typical applications in marketing or supply chain coordination. Understanding these features is key to interpreting the subsequent equilibrium results and their implications.

Fundamental Objective Asymmetry. Unlike most dyadic models where both agents seek to maximize profit (even if with different bargaining power), the players here have qualitatively different objective functions. The EDC operates as a profit-maximizer subject to a regulatory constraint (π_{EDC}), whereas the LRP firm's stated goal is to manage its operations to break even

($\pi_{LRP} = 0$). This is not merely a different parameter in the same profit function; it represents a different type of economic agent. The break-even condition for the LRP firm transforms its decision problem and its reaction function (e.g., Eq. 12), making the wholesale price mechanism to precisely cover its costs rather than to extract surplus.

Externally Imposed, System-Level Driver. The game is not initiated by bilateral negotiation over splitting a market-derived surplus. It is compelled and framed by an exogenous mandate—the energy conservation target T_E and its associated penalty/credit c_2 set by the regulator. The EDC and LRP firm are, in effect, playing a sub-game on how to most efficiently meet this third party's goal. This structure mirrors the reality of public-private partnerships and introduces an objective that is external to the dyad's direct transactional relationship, a feature absent in typical manufacturer-retailer or service provider-client models.

Specialized "Purchaser-Implementer" Roles with a Fixed-Cost Component. The model captures a clear division of labor. The EDC acts as the "funder" or "outcome purchaser," bearing the cost of incentives and the regulatory risk. The LRP firm acts as the "specialized implementer," providing the marketing platform and expertise to deliver the behavioral change. Critically, the implementer's main cost is a fixed advertising expenditure A (and a variable redemption cost), not a production or wholesale cost. This makes the pricing of its service uniquely tied to covering this upfront fixed cost at a specific volume of participation, a nuance not present in models where costs are primarily variable.

These three features—asymmetric objectives, an external system driver, and a fixed-cost implementer—define the novelty of the strategic context. The following analysis explores the equilibria and insights that emerge from applying standard game-theoretic tools to this non-standard configuration.

In the following section, we model the decision process in this B2B collaborative promotion as a sequential non-cooperative game and a cooperative game, respectively. We identify the conditions under which equilibrium solutions may exist and examine the key factors that influence the decisions of the EDC and the LRP firm. These include the EDC's penalty cost (c_2), the redemption cost of reward points (c_1), and the advertising expenses of both partners (A, B). We also analyze factors that affect household participation decisions, such as the perceived value of reward points (β), household switching costs (v_i), the average energy price difference ($p_R - p_E$), and household energy consumption levels (q_R, q_E).

4.2. *The leader-follower relationship model*

In the sequential game, there are two cases: either the EDC acts as the leader and the LRP marketing firm as the follower, or LRP marketing firm acts as the leader and the EDC follows. In the game, EDC attempts to maximize its profit, whereas the LRP marketing firm attempts to break even. Using backward induction, we obtained the equilibrium solutions.

Case 1: EDC acts as the leader, LRP marketing firm as the follower

We solve the LRP marketing firm’s problem Eq. (8) first with the EDC’s decision variable e as given. As the LRP marketing firm does not make any profit from this collaborative promotion, we have

$$\pi_{LRP}(w) = (w - c_1) \times e \times D_E - A = 0 \tag{11}$$

where $w > c_1 > 0$, $c_1 = (1 - \gamma) \times c_e$ for simplicity of the expressions.

Solve Eq. (10), we have:

$$w = \frac{A}{e \times D_E} + c_1 = \frac{A}{e \times (m_0 + m_1 \times \beta \times e)} + c_1 > 0 \tag{12}$$

From (12), we have that the LRP marketing firm’s response for setting the price (i.e., w) is an increasing function of the EDC’s decision on the number of reward points (i.e., e) given to each participating household. The decision on price w will also depend on the EDC and LRP marketing firms’ advertising expenses (i.e., A and B), redemption cost of rewards (i.e., c_1), perceived value of reward points (i.e., β), and the household switching cost (i.e., v_i).

Replace w in (9) with (12), EDC’s decision problem is:

$$\begin{aligned} \text{Max } \pi_{EDC}(e) &= p_R q_R (1 - D_E) + p_E q_E D_E - B - \left(\frac{A}{e D_E} + c_1 \right) e D_E - c_2 (T_E - D_E) \tag{13} \\ &= p_R q_R - B - c_2 T_E - A + (c_2 + p_E q_E - p_R q_R) D_E - c_1 e D_E \\ &= p_R q_R - B - c_2 T_E - A + (c_2 + p_E q_E - p_R q_R) (m_0 + m_1 \beta e) - c_1 e (m_0 + m_1 \beta e) \\ &= p_R q_R - B - A - c_2 T_E + H m_0 + (H m_1 \beta - c_1 m_0) e - c_1 m_1 \beta e^2 \\ \text{s. t. } &e > 0 \end{aligned}$$

where $H = c_2 + p_E q_E - p_R q_R$, for simplicity of the expressions. Since $\pi_{EDC}(e)$ is a concave quadratic function with respect to e , we can easily solve problem (13) and find optimal values for e and w .

Proposition 1. The non-cooperative game, where the EDC is the leader and the LRP marketing firm the follower, has a unique equilibrium (w_1^*, e_1^*) with the properties as follows

when the LRP marketing firm manages its costs and revenue by break-even (i.e., $\pi_{LRP} = 0$),

and $\frac{\beta}{c_1} > \frac{m_0}{Hm_1} > 0$.

$$e_1^* = \frac{m_1\beta H - c_1 m_0}{2c_1 m_1 \beta} = \frac{H}{2c_1} - \frac{m_0}{2m_1\beta} > 0 \tag{14}$$

$$w_1^* = \frac{A}{e_1^* \times D_E(e_1^*)} = \frac{4c_1^2 m_1 \beta A}{(Hm_1\beta)^2 - (m_0 c_1)^2} + c_1 > 0 \tag{15}$$

$$\pi_{EDC}(e_1^*) = p_R q_R - B - c_2 T_E - A + \frac{1}{2} H m_0 + \frac{m_1 \beta}{4c_1} H^2 + \frac{c_1}{4m_1 \beta} m_0^2 \tag{16}$$

$$D_E(e_1^*) = \frac{m_0}{2} + \frac{m_1 \beta H}{2c_1} \tag{17}$$

where $H = c_2 + p_E q_E - p_R q_R$, $m_0 = 1 - f_{AB} \times k_0$, $m_1 = f_{AB} \times k_1$, $f_{AB} = 1 - k_A \sqrt{A} - k_B \sqrt{B}$,

$k_1 = \sum_{i=1}^N \frac{\lambda_i}{\Delta \theta_i \Delta s_i}$, and $k_0 = \sum_{i=1}^N \lambda_i \times \frac{v_i + p_E - p_R - \theta_i \Delta s_i}{\Delta \theta_i \Delta s_i}$ for simplicity of the expressions.

$$(i) \quad \frac{\partial e_1^*}{\partial A} < 0, \frac{\partial e_1^*}{\partial B} < 0, \frac{\partial e_1^*}{\partial c_1} < 0, \frac{\partial e_1^*}{\partial k_A} < 0, \frac{\partial e_1^*}{\partial k_B} < 0, \frac{\partial e_1^*}{\partial p_R} < 0, \frac{\partial e_1^*}{\partial q_R} < 0,$$

$$\frac{\partial e_1^*}{\partial v_i} > 0, \frac{\partial e_1^*}{\partial \beta} > 0, \frac{\partial e_1^*}{\partial c_2} > 0, \frac{\partial e_1^*}{\partial p_E} > 0, \frac{\partial e_1^*}{\partial q_E} > 0.$$

$$(ii) \quad \frac{\partial w_1^*}{\partial A} > 0, \frac{\partial w_1^*}{\partial B} > 0, \frac{\partial w_1^*}{\partial c_1} > 0, \frac{\partial w_1^*}{\partial k_A} > 0, \frac{\partial w_1^*}{\partial k_B} > 0, \frac{\partial w_1^*}{\partial p_R} > 0, \frac{\partial w_1^*}{\partial q_R} > 0,$$

$$\frac{\partial w_1^*}{\partial v_i} < 0, \frac{\partial w_1^*}{\partial \beta} < 0, \frac{\partial w_1^*}{\partial c_2} < 0, \frac{\partial w_1^*}{\partial p_E} < 0, \frac{\partial w_1^*}{\partial q_E} < 0.$$

$$(iii) \quad \frac{\partial D_E}{\partial \beta} > 0, \frac{\partial D_E}{\partial c_2} > 0, \frac{\partial D_E}{\partial q_E} > 0, \frac{\partial D_E}{\partial c_1} < 0, \frac{\partial D_E}{\partial v_i} < 0, \frac{\partial D_E}{\partial q_R} < 0$$

$$\frac{\partial D_E}{\partial c_1} < 0.$$

$$(iv) \quad \frac{\partial \pi_{EDC}}{\partial v_i} < 0, \frac{\partial \pi_{EDC}}{\partial c_1} < 0, \frac{\partial \pi_{EDC}}{\partial q_E} > 0, \frac{\partial \pi_{EDC}}{\partial \beta} > 0.$$

$$(v) \quad \frac{\partial D_E}{\partial A} = \frac{k_A}{4\sqrt{A}} \left(k_0 - \frac{k_1 H \beta}{c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1}, \text{ where } \frac{k_0}{k_1} > \frac{m_0}{m_1}, \text{ and} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} \end{cases}$$

$$\frac{\partial D_E}{\partial B} = \frac{k_B}{4\sqrt{B}} \left(k_0 - \frac{k_1 H \beta}{c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1}, \text{ where } \frac{k_0}{k_1} > \frac{m_0}{m_1} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} \end{cases}$$

$$(vi) \quad \frac{\partial \pi_{EDC}}{\partial A} = \frac{(Hm_1\beta + c_1m_0)k_A}{4m_1\beta\sqrt{A}} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right) - 1, \text{ and}$$

$$\frac{\partial^2 \pi_{EDC}}{\partial A^2} = \frac{(Hm_1\beta + c_1m_0)k_A}{4m_1\beta\sqrt{A}} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right) \times \left(-\frac{1}{2\sqrt{A^3}} \right)$$

$$\text{If } \frac{\beta}{c_1} < \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right), \text{ then } \frac{\partial^2 \pi_{EDC}}{\partial A^2} < 0 .$$

$$(vii) \quad \frac{\partial \pi_{EDC}}{\partial B} = \frac{(Hm_1\beta + c_1m_0)k_B}{4m_1\beta\sqrt{B}} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right) - 1, \text{ and}$$

$$\frac{\partial^2 \pi_{EDC}}{\partial B^2} = \frac{(Hm_1\beta + c_1m_0)k_B}{4m_1\beta\sqrt{B}} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right) \times \left(-\frac{1}{2\sqrt{B^3}} \right)$$

$$\text{If } \frac{\beta}{c_1} < \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right), \text{ then } \frac{\partial^2 \pi_{EDC}}{\partial B^2} < 0 .$$

$$(viii) \quad \frac{\partial D_E}{\partial k_A} = \frac{\sqrt{A}}{2} \left(k_0 - \frac{k_1 H \beta}{c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1}, \text{ and } \frac{k_0}{k_1} > \frac{m_0}{m_1}, \text{ and} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} \end{cases}$$

$$\frac{\partial D_E}{\partial k_B} = \frac{\sqrt{B}}{2} \left(k_0 - \frac{k_1 H \beta}{c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1}, \text{ and } \frac{k_0}{k_1} > \frac{m_0}{m_1} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} \end{cases}$$

$$(ix) \quad \frac{\partial \pi_{EDC}}{\partial k_A} = \frac{(Hm_1\beta + c_1m_0)\sqrt{A}}{2m_1\beta} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right),$$

$$\frac{\partial \pi_{EDC}}{\partial k_A} \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right) \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right) \end{cases}, \text{ and}$$

$$\frac{\partial \pi_{EDC}}{\partial k_B} = \frac{(Hm_1\beta + c_1m_0)\sqrt{B}}{2m_1\beta} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1} \right),$$

$$\frac{\partial \pi_{EDC}}{\partial k_B} \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right) \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right) \end{cases}.$$

$$(x) \quad \frac{\partial \pi_{EDC}}{\partial c_2} = \frac{m_0}{2} + \frac{m_1\beta H}{2c_1} - T_E = D_E(e_1^*) - T_E \begin{cases} > 0, \text{ if } D_E(e_1^*) > T_E \\ < 0, \text{ if } D_E(e_1^*) < T_E \end{cases}.$$

$$(xi) \quad \frac{\partial D_E}{\partial p_R} = \frac{m_1}{2} \left(1 - \frac{\beta q_R}{c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{1}{q_R}, \text{ where } \frac{1}{q_R} > \frac{m_0}{Hm_1}, \text{ and} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{1}{q_R} \end{cases}$$

$$\frac{\partial D_E}{\partial p_E} = \frac{m_1}{2} \left(\frac{\beta q_E}{c_1} - 1 \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} > \frac{1}{q_E} \\ < 0, \text{ if } \frac{\beta}{c_1} < \frac{1}{q_E}, \text{ where } \frac{1}{q_E} > \frac{m_0}{Hm_1} \end{cases}.$$

$$(xii) \quad \frac{\partial \pi_{EDC}}{\partial p_R} = q_R + (m_0c_1 + m_1\beta H) \times \left(\frac{1}{2\beta} - \frac{q_R}{2c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{2\beta}{(m_0c_1 + m_1\beta H)} + \frac{1}{q_R} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{2\beta}{(m_0c_1 + m_1\beta H)} + \frac{1}{q_R} \end{cases} \text{ and}$$

$$\frac{\partial \pi_{EDC}}{\partial p_E} = (m_0c_1 + m_1\beta H) \times \left(\frac{q_E}{2c_1} - \frac{1}{2\beta} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} > \frac{1}{q_E} \\ < 0, \text{ if } \frac{\beta}{c_1} < \frac{1}{q_E}, \text{ where } \frac{1}{q_E} > \frac{m_0}{Hm_1} \end{cases}.$$

$$(xiii) \quad \frac{\partial \pi_{EDC}}{\partial q_R} = p_R \times \left(1 - \frac{m_0}{2} - \frac{m_1 \beta H}{2c_1} \right) \begin{cases} > 0, \text{ if } \frac{\beta}{c_1} < \frac{2-m_0}{m_1 H} \\ < 0, \text{ if } \frac{\beta}{c_1} > \frac{2-m_0}{m_1 H} \end{cases}$$

We obtain the equilibrium results (w_1^*, e_1^*) as in (14) and (15) with important properties as in part (i)-(xiii). At the equilibrium point, the EDC's profit and the household participation rate are as in (16) and (17). We note that the values of (w_1^*, e_1^*) depend on all model parameters except of the targeted household participation rate T_E . Therefore, changes on the value of T_E have impact solely on the value of EDC's profit.

Parts (i)-(xiii) of Proposition 1 show that at the equilibrium point (w_1^*, e_1^*) ,

- The advertising expenses A and B influence both the EDC and LRP marketing firm's decisions at the equilibrium point. The number of reward points e_1^* decreases with, and the price of reward points w_1^* increases with, the advertising expenses A and B respectively. Meanwhile, it is interesting to note that only when the ratio of perceived value and redemption cost of reward points is relatively low (e.g., $\frac{\beta}{c_1} < \frac{k_0}{Hk_1}$), will increasing advertising expenses A or B result in a higher household participation rate $D_E(e_1^*)$ and higher EDC profit π_{EDC} . Similar to this result, advertising efficacy parameters k_A and k_B may also have positive impact on household participation rate $D_E(e_1^*)$ and EDC profit $\pi_{EDC}(e_1^*)$ when the ratio $\frac{\beta}{c_1}$ is relative low (e.g., $\frac{\beta}{c_1} < \frac{k_0}{Hk_1}$). On

the other hand, when the ratio $\frac{\beta}{c_1}$ is higher than a certain threshold (e.g.,

$$\frac{\beta}{c_1} > \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1} \right),$$

increasing advertising expenses cannot improve the household participation rate and EDC profit.

- The per-unit redemption cost of reward points c_1 influences both the EDC and LRP marketing firms' decisions. When that cost increases, the EDC will reduce the number of reward points given to each participating household, whereas the LRP host firm will sell incentives to the EDC at a higher price in order to break even. The redemption cost has a negative impact on the household participation rate (i.e., higher c_1 results in a lower household participation rate).
- Regarding penalty cost c_2 , a higher c_2 will lead to a higher number of reward points e_1^* . However, the EDC's profit will increase with c_2 only if the household participation rate $D_E(e_1^*)$ is higher than the targeted participation rate T_E . On the other hand, when $D_E(e_1^*) < T_E$, the EDC's profit decreases with c_2 . This result aligns with our model setting: if the household participation rate is larger than the targeted participation rate, the EDC gains credits, whereas if the household participation rate is less than the targeted rate the EDC must pay a penalty.
- The perceived value of reward points β influences both the EDC and LRP marketing firms' decisions. It is interesting to note that the number of reward points e_1^* increases with respect to β , the perceived value of the reward points. This result reveals that the EDC is willing to purchase more points and give those points to participating households if the reward points are more attractive to households; in other words, the LRP has higher effectiveness. One reason is that compared with the effectiveness of the LRP, the increased cost for the EDC to purchase more points is relatively small when LRP marketing firm manages its cost and revenue by break-even. The other reason is that the EDC gains more negotiation power if they purchase more points from the LRP marketing firm.
- Increases in β also result in a higher household participation rate and higher profit for the EDC.
- Regarding the household switching cost v_i , it is not surprising to observe that the number of reward points e_1^* increases with v_i . However, the household participation rate and the EDC's profit decrease with v_i .

- Regarding average energy prices p_E and p_R , the number of reward points e_1^* decreases with p_R , but increases with p_E . This result suggests that when there is a bigger difference between the average energy prices before and after a household joining an energy-conservation program (i.e. $(p_R - p_E)$ is higher), the EDC will reduce the number of reward points given to each participating household. However, the impact of the price difference $(p_R - p_E)$ on the household participation rate and the EDC's profit will depend on the ratio of perceived value and redemption cost of reward points (i.e., $\frac{\beta}{c_1}$). If the ratio is relatively small (e.g., $\frac{\beta}{c_1} < \frac{1}{q_R}$), the household participation rate and EDC profit will increase with respect to $(p_R - p_E)$. On the other hand, when the ratio is over a certain threshold (e.g., $\frac{\beta}{c_1} > \max \left\{ \frac{1}{q_E}, \frac{m_0}{Hm_1}, \sqrt{\left(\frac{m_0}{2Hm_1} - \frac{1}{2q_R} - \frac{1}{Hm_1} \right)^2 + 4} \right\}$), the positive impact of $(p_R - p_E)$ on the household participation rate and EDC profit will diminish.
- Regarding energy consumptions q_E and q_R , it is interesting to note that in most cases, higher energy saving $(q_R - q_E)$ results in a smaller number of reward points e_1^* , a lower household participation rate $D_E(e_1^*)$, and lower EDC profit $\pi_{EDC}(e_1^*)$. This result suggests that energy saving cannot create a positive impact on the household participation rate and EDC profit.

Overall, among all those key factors, we find that both the perceived value β and the redemption cost of LRP incentives c_1 play a key role in determining the impact of advertising expenses and average energy price difference $p_R - p_E$ on household participation and system-wide profit. They also are indicators that reflect the effectiveness and efficiency of LRP operations. Both the EDC and the LRP marketing firm should pay attention to those factors and their impact on household participation and system-wide profit. On the other hand, our findings also suggest that when promoting the energy-conservation program in this cooperative

promotion, the EDC and the LRP marketing firm may not need to focus on energy saving ($q_R - q_E$) as it does not affect the household participation rate.

Case 2: LRP marketing firm acts as the leader, EDC as the follower

We solve EDC problem Eq. (8) first with the LRP marketing firm’s decision variable w as given.

$$\begin{aligned} \text{Max } \pi_{EDC}(e) &= p_R q_R (1 - D_E) + p_E q_E D_E - B - w \times e \times D_E - c_2 (T_E - D_E) \quad (18) \\ &= p_R q_R - B - c_2 T_E + (c_2 + p_E q_E - p_R q_R) D_E - w \times e \times D_E \\ &= p_R q_R - B - c_2 T_E + (c_2 + p_E q_E - p_R q_R) (m_0 + m_1 \beta e) - w \times e \times (m_0 + m_1 \beta e) \\ &= p_R q_R - B - c_2 T_E + (c_2 + p_E q_E - p_R q_R) m_0 + [(c_2 + p_E q_E - p_R q_R) m_1 \beta - w m_0] e \\ &\quad - w m_1 \beta e^2 \end{aligned}$$

s.t.

$$e > 0$$

We have:

$$e_2^* = \frac{m_1 \beta H - w m_0}{2 w m_1 \beta} = \frac{H}{2w} - \frac{m_0}{2m_1 \beta} > 0 \quad (19)$$

Replace e in Eq. (8) with (19), the LRP marketing firm’s decision problem is:

$$\pi_{LRP}(w) = (w - c_1) \times \left(\frac{H}{2w} - \frac{m_0}{2m_1 \beta} \right) \times \left(m_0 + m_1 \beta \left(\frac{H}{2w} - \frac{m_0}{2m_1 \beta} \right) \right) - A = 0 \quad (20)$$

As the LRP marketing firm does not need to make any profit from this collaborative promotion, from Eq. (20), we have:

$$w^3 + \frac{4A m_1 \beta - c_1 m_0^2}{m_0^2} \times w^2 - \frac{m_1^2 \beta^2 H^2}{m_0^2} \times w + \frac{c_1 m_1^2 \beta^2 H^2}{m_0^2} = 0 \quad (21)$$

Eq. (21) is a cubic function of w . Due to the computational complexity, we cannot find closed forms for the equilibrium solutions of Eq. (20) and Eq. (19). If an equilibrium solution (w_2^*, e_2^*) exists in Case 2, then that solution has the following proposition.

Proposition 2. The Stackelberg game, where the LRP marketing firm acts as the leader and manages its costs and revenue by break-even, and the EDC as the follower, has a unique equilibrium (w_2^*, e_2^*) if and only if:

$$\begin{aligned} (w_2^*)^3 + \frac{4A m_1 \beta - c_1 m_0^2}{m_0^2} \times (w_2^*)^2 - \frac{m_1^2 \beta^2 H^2}{m_0^2} \times w_2^* + \frac{c_1 m_1^2 \beta^2 H^2}{m_0^2} &= 0 \quad (22) \\ \text{and } 0 < w_2^* < \frac{m_1 \beta H}{m_0}. \end{aligned}$$

Thus,

$$e_2^* = \frac{H}{2w_2^*} - \frac{m_0}{2m_1\beta} > 0 \tag{23}$$

where $H = c_2 + p_E q_E - p_R q_R$, $m_0 = 1 - f_{AB} \times k_0$, $m_1 = f_{AB} \times k_1$, $f_{AB} = 1 - k_A \sqrt{A} - k_B \sqrt{B}$,

$$k_1 = \sum_{i=1}^N \frac{\lambda_i}{\Delta \theta_i \Delta s_i}, \text{ and } k_0 = \sum_{i=1}^N \lambda_i \times \frac{v_i + p_E - p_R - \theta_i \Delta s_i}{\Delta \theta_i \Delta s_i} \text{ for simplicity of the expressions.}$$

At the equilibrium point (w_2^*, e_2^*) , the EDC’s optimal profit is as in (24) and the household participation rate is as in (25):

$$\pi_{EDC}(e_2^*) = p_R q_R - B - c_2 T_E - A + H m_0 + (m_1 \beta H - c_1 m_0) e_2^* - w_2^* m_1 \beta (e_2^*)^2 \tag{24}$$

$$D_E(e_2^*) = \frac{m_0}{2} + \frac{m_1 \beta H}{2w_2^*} \tag{25}$$

In Table 1, we list the equilibrium solutions in both Case 1 and Case 2. Obviously as $w_2^* > c_1 > 0$, we find that in Case 1 (i.e., EDC as the leader in the sequential game), the amount of incentives (i.e., e_1^*) given to each participating household will always be more than (i.e., e_2^*) in Case 2 (i.e., LRP marketing firm as the leader in the game). Thus, the household participation rate (i.e., $D_E(e_1^*)$) in Case 1 is also higher than (i.e., $D_E(e_2^*)$) in Case 2, and the LRP marketing firm will set lower incentive price in Case 1 than in Case 2 (i.e., $w_1^* < w_2^*$). Furthermore, the EDC’s profit in Case 1 will be no less than in Case 2 (i.e., $\pi_{EDC}(e_1^*) \square \pi_{EDC}(e_2^*)$). These results reveal that in the sequential game, the EDC will gain more benefits in Case 1 (e.g., achieve higher profit and household participation rate) than in Case 2.

Table 1: Equilibrium Solutions in the Sequential Game

	Case 1: EDC acts as the leader, LRP marketing firm as the follower	Case 2: LRP marketing firm acts as the leader, EDC as the follower
EDC's decision: ordering quantity of reward points	$e_1^* = \frac{H}{2c_1} - \frac{m_0}{2m_1\beta} > 0$ exists when $0 < \frac{\beta}{c_1} < \frac{m_0}{m_1H}$	$e_2^* = \frac{H}{2w_2^*} - \frac{m_0}{2m_1\beta} > 0$ exists when $0 < w_2^* < \frac{m_1\beta H}{m_0}$
LRP marketing firm's decision: wholesale price of reward points	$w_1^* = \frac{A}{e_1^* \times D_E(e_1^*)} + c_1 > c_1 > 0$	$w_2^* = \frac{A}{e_2^* \times D_E(e_2^*)} + c_1 > c_1 > 0$
Household participation rate	$D_E(e_1^*) = \frac{m_0}{2} + \frac{m_1\beta H}{2c_1} > 0$	$D_E(e_2^*) = \frac{m_0}{2} + \frac{m_1\beta H}{2w_2^*} > 0$
EDC's profit	$\pi_{EDC}(e_1^*) = p_R q_R - B - c_2 T_E - A + H m_0 + (m_1\beta H - c_1 m_0) e_1^* - w_1^* m_1\beta (e_1^*)^2$	$\pi_{EDC}(e_2^*) = p_R q_R - B - c_2 T_E - A + H m_0 + (m_1\beta H - c_1 m_0) e_2^* - w_1^* m_1\beta (e_2^*)^2$

4.3 The cooperative relationship model

In the cooperative game, both the EDC and the LRP marketing firm agree to make decisions that maximize system profit (i.e., joint profit maximization). The system profit is described by Eq. (10). We hence have the following optimization problem:

$$\begin{aligned} \text{Max } \pi_{LRP+EDC}(e) &= p_R q_R (1 - D_E) + p_E q_E D_E - B - c_1 e D_E - A - c_2 (T_E - D_E) \quad (26) \\ &= p_R q_R - B - c_2 T_E - A + (c_2 + p_E q_E - p_R q_R) D_E - c_1 e D_E \end{aligned}$$

s. t. $e > 0$

We have a unique solution (\bar{e}) for problem (26).

$$\bar{e} = \frac{m_1\beta H - c_1 m_0}{2c_1 m_1\beta} = \frac{H}{2c_1} - \frac{m_0}{2m_1\beta} > 0 \quad (27)$$

Obviously, this solution is equal to the solution in the non-cooperative sequential game Case 1.

$$\bar{e} = e_1^* \quad (28)$$

As such, the household participation rates in the cooperative game and in the non-cooperative game Case 1 are the same.

$$D_E(\bar{e}) = D_E(e_1^*) = \frac{m_0}{2} + \frac{m_1\beta H}{2c_1} \quad (29)$$

As the LRP marketing firm does not take any profit away from the system, the EDC will take all the profit and in turn carry all the costs incurred. In this case, maximizing the system profit in the cooperative game is equivalent to maximizing the EDC's profit in the non-cooperative

game where EDC acts as the leader. Therefore, EDC’s optimal profit in the non-cooperative game Case 1 is equal to the system’s optimal profit in the cooperative game.

$$\pi_{LRP+EDC}(\bar{e}) = \pi_{EDC}(e_1^*) = p_R q_R - B - c_2 T_E - A + \frac{Hm_0}{2} + \frac{m_1 \beta H^2}{4c_1} + \frac{c_1 m_0^2}{4m_1 \beta} \tag{30}$$

Meanwhile the system’s total promotion and advertising cost is as follows:

$$C_{LRP+EDC}(\bar{e}) = A + \frac{m_1 \beta H^2}{4c_1} - \frac{c_1 m_0^2}{4m_1 \beta} + B \tag{31}$$

Some important observations regarding $\pi_{LRP+EDC}(\bar{e})$ and $C_{LRP+EDC}(\bar{e})$ are listed in Table 2.

Table 2: Cooperative Game Results

Parameters				
	\bar{e}	$D_E(\bar{e})$	$\pi_{LRP+EDC}$	$C_{LRP+EDC}$
EDC’s advertising spending: A	-	+	+/- ⁽¹⁾	+/- ⁽⁴⁾
LRP marketing firm’s advertising spending: B	-	+	+/- ⁽²⁾	+/- ⁽⁵⁾
Efficacy of EDC’s advertising: k_A	-	+	+/-	-
Efficacy of LRP marketing firm’s advertising: k_B	-	+	+/-	-
LRP marketing firm’s actual redemption cost: c_I	-	-	-	-
EDC’s penalty cost: c_2	+	+	+/- ⁽³⁾	+
household switching cost: v_i	+	-	-	+
The perceived value of reward points: β	+	+	+	+
Average energy price in regular energy program: p_R	-	+/- ⁽⁶⁾	+/- ⁽⁸⁾	-
Average energy price in energy-conservation program p_E	+	+/- ⁽⁷⁾	+/- ⁽⁹⁾	+
Energy consumption in regular energy program: q_R	-	-	+/- ⁽¹⁰⁾	-
Energy consumption in energy-conservation program: q_E	+	+	+	+

“+” (or “-”) indicates that when the value of the parameter increases, the values of the decision variable \bar{e} as well as household participation rate $D_E(\bar{e})$, system profit $\pi_{LRP+EDC}$, and overall system-wide promotion and advertising cost $C_{LRP+EDC}$ will also increase (or

decrease). “+/-” indicates that when the value of the parameter increases, the values of \bar{e} , $D_E(\bar{e})$, $\pi_{LRP+EDC}$, or $C_{LRP+EDC}$ may increase, or decrease (see Appendix for more details about notes (1)-(10) in the table).

In Table 2, we list the impact of key parameters on decision variable e , system-wide profit $\pi_{LRP+EDC}$ and system-wide cost $C_{LRP+EDC}$. Although most of the results in Table 2 are the same as the results in non-cooperative game Case 1, one thing worth mentioning is that at the optimal

point \bar{e} , in the case that $\frac{m_0}{Hm_1} < \frac{\beta}{c_1} < \frac{k_0}{Hk_1} + \left(\frac{k_0}{Hk_1} + \frac{m_0}{Hm_1}\right)$, system profit $\pi_{LRP+EDC}(\bar{e})$ reaches

its maximum value when advertising expense A equals to $\left[\frac{k_A(Hm_1\beta + c_1m_0)}{4m_1\beta} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1}\right)\right]^2$, or advertising expense B equals to

$\left[\frac{k_B(Hm_1\beta + c_1m_0)}{4m_1\beta} \times \left(k_0 - \frac{(Hm_1\beta - c_1m_0)k_1}{2m_1c_1}\right)\right]^2$; whereas system total promotion and

advertising cost $C_{LRP+EDC}(\bar{e})$ reaches its minimum value when advertising expenses A and B

equal to $\left[\frac{H^2\beta k_1 k_A}{8c_1} + \frac{m_0 c_1 k_A (m_0 k_1 + 2m_1 k_0)}{8m_1^2 \beta}\right]^2$ and $\left[\frac{H^2\beta k_1 k_B}{8c_1} + \frac{m_0 c_1 k_B (m_0 k_1 + 2m_1 k_0)}{8m_1^2 \beta}\right]^2$

respectively. This result suggests that at the equilibrium point ($\bar{e} = \frac{H}{2c_1} - \frac{m_0}{2m_1\beta} > 0$), when the

ratio $\frac{\beta}{c_1}$ is under a certain threshold (e.g., $\frac{\beta}{c_1} < \frac{k_0}{Hk_1}$), determining an appropriate advertising

budget may not only help increase household participation rate, but also maximize the overall system profit, and minimize the overall advertising and promotion costs.

5. Discussion of Results and Implications

In this section, we will discuss the key findings from the model results obtained in Section 4 and reveal useful insights into the design and implementation of a LRP for household energy-conservation initiatives.

5.1. Summary of Key Findings

Based on the equilibrium analysis in Section 4, our model yields several important insights for designing and managing LRP-based energy conservation partnerships:

The Leadership Advantage: In the non-cooperative game, when the EDC acts as the Stackelberg leader and the LRP firm operates on a break-even basis, the resulting system-wide profit and household participation rate can match those of a fully cooperative scenario. This identifies an implementable (non-cooperative) structure that achieves cooperative efficiency.

The Central Role of the Value-to-Cost Ratio (β / c_1): The ratio of the perceived value of reward points (β) to their redemption cost (c_1) is a critical contingency factor. It determines the effectiveness of other policy levers, such as advertising spending and energy price differentials, on household participation and system profit.

The Limits of Incentives: Reward points cannot always overcome high household switching costs (v_i). Increasing the incentive level does not fully mitigate the negative impact of switching costs on participation, highlighting the need to reduce these costs through complementary program design (e.g., better information, easier enrollment).

The Insignificance of Average Savings: Contrary to intuition, the average per-household energy saving ($q_R - q_E$) does not significantly influence the household participation rate in our model, suggesting that promotional messaging should focus on the immediacy and tangibility of rewards rather than on long-term bill savings.

5.2. Detailed Analysis and Managerial Implications

According to the equilibrium solutions (see in Eqs. (14)-(17), (22)-(25), and (27)-(31)), the key findings are summarized as follows.

5.2.1 Achieving System Efficiency through EDC Leadership

Finding 1: When the LRP marketing firm operates on a break-even basis, maximizing the profit of the Energy Distribution Company (EDC) in a non-cooperative sequential game where the EDC is the leader is equivalent to maximizing the total system profit in a cooperative game. This suggests that for promoting household energy conservation, the EDC should act as the leader, not the follower, and decide the optimal incentive level (reward points per household). In this case, the EDC achieves its highest profit, the highest household participation rate, and system-wide optimum.

Supporting Analytical Result: This conclusion stems from comparing the equilibrium solutions of the non-cooperative leader-follower games (Section 4.2) and the cooperative game (Section 4.3). Comparing two scenarios' optimal number of reward points shows that $\bar{e} = e_1^*$ (E.q. 28). Therefore, the EDC's profit-maximizing incentive level as a leader is identical to the

system-optimal level. Consequently, the household participation rates are equal, and the EDC's maximum profit in the leader role equals the maximum system profit in cooperation (E.q. 30). And Table 1 summarizes that when the LRP firm leads (Case 2), the incentive price w_2^* is higher. This leads to a lower optimal incentive quantity e_2^* , a lower household participation rate, and a lower (or at best, no higher) profit for the EDC compared to Case 1. This reinforces the advantage of the EDC being the leader.

Theoretical and Practical Implications: From a theoretical perspective: The finding demonstrates that under specific conditions (LRP break-even operation), a particular non-cooperative game structure (EDC as Stackelberg leader) can achieve the same efficient outcome as a fully cooperative game. This provides a theoretical basis for efficient coordination in public-private partnerships where partners have asymmetric objectives. From a practical perspective, there are three main implications. **Leadership Structure:** In forming partnerships for energy conservation programs, the EDC (the entity bearing the conservation target and core costs) should be contractually designated as the decision-maker for the incentive level. **Performance Expectation:** This structure aligns the EDC's self-interest with the system's goal. The EDC is intrinsically motivated to choose the system-optimal incentive level, leading to high participation without needing complex profit-sharing contracts. **Contract Design:** Agreements should explicitly grant the EDC the leadership role in deciding the incentive level, thereby institutionalizing this efficient equilibrium and avoiding suboptimal outcomes from ambiguous decision rights.

In summary, Finding 1 not only mathematically proves the efficiency of EDC leadership but also offers crucial guidance for designing the governance structure of such energy conservation initiatives.

5.2.2 The Pivotal Value-to-Cost Ratio and Its Managerial Leverage

Finding 2: The ratio of the perceived value to the redemption cost of reward points (β/c_1) is a critical contingency factor. It fundamentally moderates the impact of other operational variables, such as advertising spending and energy price differentials, on household participation and system profit. Moreover, a trade-off exists in the EDC's decision between advertising expenses and the number of reward points offered. Importantly, increasing the number of reward points cannot fully compensate for high household switching costs, which directly suppress participation.

Supporting Analytical Result: This finding is primarily supported by the comparative statics and sensitivity analyses derived from the equilibrium solution in Case 1 (EDC as leader), as

detailed in Proposition 1 (Parts (i)-(xiii)). Trade-off and Moderating Role: At equilibrium, increasing advertising spending (A or B) causes the EDC to offer fewer reward points (i.e., e_1^* decreases) and the LRP firm to charge a higher price (i.e., w_1^* increase) (Parts (i)-(ii) of Proposition 1). This reveals a substitutable relationship. However, the impact of advertising on the participation rate D_E is positive only if the ratio β/c_1 is below a specific threshold, i.e., $\beta/c_1 < k_0/(Hk_1)$ (Part (v)). Beyond this threshold, more advertising yields diminishing returns. Limit of Incentives vs. Switching Costs: The analysis shows that while the EDC responds to higher household switching costs v_i by offering more points ($\partial e_1^*/\partial v_i > 0$), the participation rate nevertheless decreases ($\partial D_E/\partial v_i < 0$) (Parts (i) and (iii)). This indicates that financial incentives alone cannot neutralize the disincentive created by high non-monetary adoption barriers.

Theoretical and Practical Implications: This result establishes β/c_1 as a key moderating variable in models of incentivized behavior change. It defines the boundary conditions for the effectiveness of other instruments like advertising, enriching the theory of marketing-operations coordination. Strategic Budget Allocation: Managers must evaluate β/c_1 to decide the optimal mix between spending on incentives versus advertising. A high ratio favors using incentives; a low ratio necessitates greater reliance on advertising to drive participation. Program Design Priority: Since incentives cannot compensate for high switching costs, program designers must directly reduce these costs by simplifying processes, providing support, and improving convenience to lower the time, effort, and hassle for households. Partner Performance Metric: The EDC should treat β/c_1 as a Key Performance Indicator (KPI) for the LRP partnership. Collaborating to enhance the perceived value (β) of points or reduce their redemption cost (c_1) improves program efficiency and strategic flexibility.

In summary, Finding 2 provides a clear decision framework: manage the value-to-cost ratio, balance incentive and advertising spending based on it, and address the switching costs directly through program design, not just through larger incentives.

5.2.3 Rethinking Program Strategy: The Centrality of the Value-to-Cost Ratio

Finding 3: The ratio of the perceived value to the redemption cost of reward points (β/c_1) is the decisive lever controlling overall program efficiency. It governs whether increasing advertising budgets or energy price differentials will improve household participation and profit. A high β/c_1 ratio allows for reduced advertising spend without sacrificing performance, whereas a low ratio makes advertising and price incentives more critical.

Supporting Analytical Result: This finding is a synthesis and extension of the sensitivity results from the equilibrium solution (Proposition 1), specifically concerning the conditional effects of advertising (A, B) and price differences ($p_R - p_E$). Proposition 1(v) and (x) show that the impact of advertising spending (A, B) and its efficacy ($k_A - k_B$) on the household participation rate (D_E) depends entirely on the ratio β / c_1 relative to the threshold $k_0 / (Hk_1)$. Similarly, Proposition 1(xii) demonstrates that the influence of the average energy price difference ($p_R - p_E$) on participation is also moderated by β / c_1 , relative to thresholds involving household energy consumption ($1/q_R, 1/q_E$). The effects on the EDC's profit (π_{EDC}), which equals system profit in the leader case, follow the same contingent pattern. Proposition 1(vi), (vii), and (xii) show that the profit's sensitivity to changes in A, B, p_R , and p_E reverses depending on whether β / c_1 is above or below specific critical values.

Theoretical and Practical Implications: This finding establishes the ratio β / c_1 as a central moderating variable in models of incentivized behavior change. It defines the conditions under which other instruments, like advertising or price signals, are effective, thereby advancing the theory of marketing-operations coordination. Primary Management Focus: Managers should treat improving the ratio β / c_1 as a core goal. This involves enhancing the attractiveness of reward points (β) and reducing their redemption cost (c_1). Dynamic Strategy Setting: The value of β / c_1 dictates the optimal promotional mix. A high ratio justifies a focus on incentives, potentially allowing for lower advertising spend. A low ratio necessitates greater reliance on advertising and price discounts to achieve targets. Integrated Decision-Making: Decisions on advertising budgets, pricing, and incentive levels should be made jointly, guided by the current β / c_1 ratio. This enables evidence-based resource allocation across the promotional toolkit.

5.2.4 Rethinking the Message: Rewards over Savings

Finding 4: Unlike the average energy price difference that may complement the reward points to improve the household participation rate and system-wide profit, average energy saving (i.e., $(q_R - q_E)$) does not affect household participation. Contrary to conventional emphasis on long-term bill savings, the average per-household energy saving $(q_R - q_E)$ does not significantly influence the household participation decision in our model. The immediate and tangible value of reward points, alongside price signals and advertising, are the primary drivers. This suggests promotional messaging should prioritize the appeal of instant rewards over projections of future energy cost savings.

Supporting Analytical Result: This finding is derived from the household decision model and the resulting sensitivity analysis of the equilibrium solution. The household's surplus function for joining the conservation program ($U_{E(i)}$, E.q. 1) does not include a term for the future monetary value of energy savings $(q_R - q_E)$. The decision is myopic, based on immediate perceived value. And the comparative statics for the household participation rate D_E with respect to q_E and q_R (Proposition 1(iii)) show opposing effects ($\partial D_E / \partial q_E > 0, \partial D_E / \partial q_R < 0$). More importantly, the subsequent discussion in Section 4.2 concludes that energy saving cannot create a positive impact on the household participation rate, confirming that the net effect of savings is not a statistically significant driver in the model equilibrium.

Theoretical and Practical Implications: This finding provides a formal model-based confirmation of a behavioral insight: consumers place greater weight on immediate, salient rewards than on delayed, probabilistic savings when making conservation decisions. It integrates concepts like hyperbolic discounting into the analysis of energy program design. **Message Strategy:** The core promotional message for such programs should be reframed. Lead with the reward, not the long-term savings. Communications should emphasize “Get reward now for this action” rather than “Save \$X per year.”

Program Justification: This finding validates the use of Loyalty Reward Programs (LRPs) as a central tool. LRPs are effective precisely because they provide the immediate, tangible incentives that align with actual household decision-making, unlike messaging focused solely on future utility bill reductions. **Information Role:** While long-term savings estimates are not the primary motivator, they can still play a secondary, supportive role in program communications (e.g., to enhance credibility or provide a rationale), but they should not be the headline.

6. Summary and Future Research

This study develops a game-theoretic model to investigate the coordination between an energy distribution company (EDC) and a loyalty reward program (LRP) marketing firm in a partnership aimed at promoting household energy conservation. Our analysis, which considers a break-even LRP operator and a profit-maximizing EDC subject to a regulatory target, yields the following principal findings and implications.

6.1 Key Findings

Optimal Partnership Structure: When the LRP firm operates on a break-even basis, the EDC acting as the leader in a non-cooperative, sequential game can achieve system-wide profit and household participation rates equivalent to those of a fully cooperative partnership. This identifies an implementable governance structure that achieves coordination efficiency without requiring complex, fully integrated cooperation.

Central Role of Incentive Efficiency: The ratio of the perceived value to the redemption cost of LRP incentives is a critical contingency factor. It determines the effectiveness of other operational levers:

Advertising & Pricing: Increasing advertising spending or the energy price difference between regular and conservation plans boosts participation and profit only when this ratio is below a certain threshold. When incentive efficiency is high, the marginal benefit of additional advertising or price differentiation diminishes.

Managerial Leverage: This finding shifts the focus from simply spending more on promotions to improving the fundamental appeal and cost-effectiveness of the rewards themselves.

Limits of Incentives and Other Drivers: Our model reveals two important boundary conditions. Reward points cannot fully compensate for high household switching costs (e.g., time, effort, inconvenience), underscoring the need for programs to also simplify participation processes. The average per-household energy saving has no significant impact on the participation rate, suggesting that promotional messaging should emphasize immediate, tangible rewards rather than long-term, uncertain bill savings.

6.2 Managerial and Policy Implications

For EDCs and regulators, the results advocate for partnership designs where the EDC leads in setting the incentive level. This structure harnesses the EDC's profit motive and regulatory accountability to naturally align with system-wide goals.

For program managers (both EDC and LRP firm), the analysis provides a clear decision framework: First, work to increase the incentive value-cost ratio by making points more attractive or redemption cheaper. Second, calibrate advertising budgets and pricing strategies based on this achieved ratio. Blind increases in spending are inefficient if the core incentive is not compelling or cost-effective.

For program design, efforts must extend beyond financial incentives. Proactively reducing household switching costs through better information, streamlined enrollment, and supportive services is crucial for broadening participation.

6.3 Limitations and Future Research

Although our paper contains interesting insights on the implementations of LRP for household energy-conservation, they must be interpreted within the context of certain limitations. First of all, our study is based on a real-world cooperation promotion application in which the energy firm is a profit maker whereas the LRP marketing firm manages its revenue and costs on a break-even basis. As researchers (e.g., De T'Serclaes 2007, Jamasb and Pollitt 2011) have pointed out, public-private partnerships (PPP) play an increasingly important role in energy-demand management. PPP may offer better opportunities for energy-conservation initiatives in terms of relevance, flexibility, impact, and sustainability because they enable the integration of resources from private and public sectors, combining the strengths of both. The models developed in this paper can be extended and modified to study the partnership between profit and not-for-profit organizations for energy-conservation and other social change programs. Second, we use a linear function to model the impact of incentives on households' participation preferences. We assume that the impact is the same across all household segments. In future research, other approaches to model the impact of incentives, as well as differentiation of the impacts among household segments should be explored. Another possible extension of this study is to consider other types of households' utility functions, especially when households face multiple discrete-continuous choice situations in energy-conservation promotion.

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Appendix

Table A1

Parameters	Decision Variables and Results				
	e_1^*	w_1^*	D_E	$\pi_{EDC} = \pi_{LRP+EDC}$	$C_{LRP+EDC}$
A	-	+	+	+/- ⁽¹⁾	+/- ⁽⁴⁾
c_1	-	+	-	-	-
v_i	+	-	+	-	+
β	+	-	+	+	+
k_A	-	+	+	+	-
k_B	-	+	+	+	-
B	-	+	+	+/- ⁽²⁾	+/- ⁽⁵⁾
c_2	+	-	+	+/- ⁽³⁾	+
p_R	-	+	+/- ⁽⁶⁾	+/- ⁽⁸⁾	-
p_E	+	-	+/- ⁽⁷⁾	+/- ⁽⁹⁾	+
q_R	-	+	-	-	-
q_E	+	-	+	+	+

In **Table A1**, “+” (or “-”) indicates that when the value of the parameter increases, the values of the decision variables as well as D_E , π_{EDC} , and $C_{LRP+EDC}$ will also increase (or decrease). “+/-” indicates that when the value of the parameter increases, the values of the decision variables may increase, or decrease.

In Table A1 note(1) and note(2), based on Eq. (15) and (29), we have:

$$\frac{d\pi_{LRP+EDC}}{dA} = \frac{d\pi_{EDC}}{dA} = \frac{k_A}{4\sqrt{A}} \left(H + \frac{c_1 K}{k_1 \beta} \right) - 1 \tag{A1}$$

$$\frac{d^2\pi_{LRP+EDC}}{dA^2} = \frac{d^2\pi_{EDC}}{dA^2} = -\frac{k_A H}{8\sqrt{A^3}} - \frac{K}{2\sqrt{A^3}} - \frac{k_A^2 c_1}{8k_1 \beta A} < 0 \tag{A2}$$

$$\frac{d\pi_{LRP}}{dB} = \frac{d\pi_{LRP+EDC}}{dB} = \frac{d\pi_{EDC}}{dB} = \frac{k_B}{4\sqrt{B}} \left(H + \frac{c_1 K}{k_1 \beta} \right) - 1 \tag{A3}$$

$$\frac{d^2\pi_{LRP}}{dB^2} = \frac{d^2\pi_{LRP+EDC}}{dB^2} = \frac{d^2\pi_{EDC}}{dB^2} = -\frac{k_B H}{8\sqrt{B^3}} - \frac{K}{2\sqrt{B^3}} - \frac{k_B^2 c_1}{8k_1\beta A} < 0 \tag{A4}$$

(A1) and (A2) show that when $A = \left(\frac{k_A k_1 \beta H + c_1 k_0 + c_1 k_B \sqrt{B}}{4k_1\beta - c_1 k_A} \right)^2$, the EDC's profit at equilibrium point, as well as the system profit will reach their maximum values. When $A < \left(\frac{k_A k_1 \beta H + c_1 k_0 + c_1 k_B \sqrt{B}}{4k_1\beta - c_1 k_A} \right)^2$, the EDC's profit and the system profit increase with A , whereas when $A > \left(\frac{k_A k_1 \beta H + c_1 k_0 + c_1 k_B \sqrt{B}}{4k_1\beta - c_1 k_A} \right)^2$, the EDC's profit and the system profit decrease with A .

Similarly, (A3) and (A4) show that when $B = \left(\frac{k_B k_1 \beta H + c_1 k_0 + c_1 k_A \sqrt{A}}{4k_1\beta - c_1 k_B} \right)^2$, the EDC's profit at equilibrium point and the system profit will reach their maximum values. When $B < \left(\frac{k_B k_1 \beta H + c_1 k_0 + c_1 k_A \sqrt{A}}{4k_1\beta - c_1 k_B} \right)^2$, the EDC's profit and the system profit increase with A , whereas when $B > \left(\frac{k_B k_1 \beta H + c_1 k_0 + c_1 k_A \sqrt{A}}{4k_1\beta - c_1 k_B} \right)^2$, the EDC's profit and the system profit decrease with A .

In Table A1 note (3), based on Eq. (15) and (29), we have:

$$\frac{d\pi_{LRP+EDC}}{dc_2} = \frac{K}{2} + \frac{k_1\beta H}{2c_1} - T_E = D_E(e^*) - T_E \tag{A5}$$

$$\frac{d^2\pi_{LRP+EDC}}{dc_2^2} = \frac{k_1\beta}{2c_1} > 0 \tag{A6}$$

(A5) and (A6) show that when $D_E(e^*) > T_E$, the EDC's profit and the system profit will increase with c_2 . On the other hand, when $D_E(e^*) < T_E$, the EDC's profit and the system profit will decrease with c_2 .

This result aligns with our model setting; if the household participation rate is larger than the targeted participation rate, the EDC gains credits, whereas if the household participation rate is less than the targeted rate, the EDC has to pay a penalty.

In Table A1 note (4) and note (5), based on Eq. (30), we have:

$$\frac{dC_{LRP+EDC}}{dA} = 1 - \frac{k_A c_1 (k_0 + k_A \sqrt{A} + k_B \sqrt{B})}{4k_1\beta\sqrt{A}} \tag{A7}$$

$$\frac{d^2C_{LRP+EDC}}{dA^2} = \frac{k_A c_1 (k_0 + k_B \sqrt{B})}{8k_1 \beta \sqrt{A}^3} > 0 \tag{A8}$$

$$\frac{dC_{LRP+EDC}}{dB} = 1 - \frac{k_B c_1 (k_0 + k_A \sqrt{A} + k_B \sqrt{B})}{4k_1 \beta \sqrt{B}} \tag{A9}$$

$$\frac{d^2C_{LRP+EDC}}{dB^2} = \frac{k_B c_1 (k_0 + k_A \sqrt{A})}{8k_1 \beta \sqrt{B}^3} > 0 \tag{A10}$$

(A7) and (A8) show that when $4k_1\beta - c_1k_A^2 > 0$ and $A = \left(\frac{k_A c_1 (k_0 + k_B \sqrt{B})}{4k_1 \beta - c_1 k_A^2} \right)^2$, $C_{LRP+EDC}$ will reach

its minimum value. $C_{LRP+EDC}$ will increase with A when $A > \left(\frac{k_A c_1 (k_0 + k_B \sqrt{B})}{4k_1 \beta - c_1 k_A^2} \right)^2$, whereas when

$A < \left(\frac{k_A c_1 (k_0 + k_B \sqrt{B})}{4k_1 \beta - c_1 k_A^2} \right)^2$, $C_{LRP+EDC}$ will decrease with A . Similarly, (A9) and (A10) show that when

$4k_1\beta - c_1k_A^2 > 0$ and $B = \left(\frac{k_B c_1 (k_0 + k_A \sqrt{A})}{4k_1 \beta - c_1 k_B^2} \right)^2$, $C_{LRP+EDC}$ will reach its minimum value. $C_{LRP+EDC}$

will increase with B when $B > \left(\frac{k_B c_1 (k_0 + k_A \sqrt{A})}{4k_1 \beta - c_1 k_B^2} \right)^2$, whereas when $B < \left(\frac{k_B c_1 (k_0 + k_A \sqrt{A})}{4k_1 \beta - c_1 k_B^2} \right)^2$,

$C_{LRP+EDC}$ will decrease with B .

In Table A1 note (6) and note (7), based on Eq. (16), we have:

$$\frac{d(D_E)}{dp_E} = \frac{1}{2} \times \frac{dK}{dp_E} + \frac{k_1 \beta}{2c_1} \times \frac{dH}{dp_E} = \frac{k_1}{2} \left(\frac{\beta q_E}{c_1} - 1 \right) \tag{A11}$$

$$\frac{d(D_E)}{dp_R} = \frac{1}{2} \times \frac{dK}{dp_R} + \frac{k_1 \beta}{2c_1} \times \frac{dH}{dp_R} = \frac{k_1}{2} \left(1 - \frac{\beta q_R}{c_1} \right) \tag{A12}$$

(A11) and (A12) show that if $\beta > \frac{c_1}{q_E}$, $\frac{d(D_E)}{dp_E} > 0$, the household participation rate increases with the

energy price p_E ; whereas if $\beta < \frac{c_1}{q_E}$, $\frac{d(D_E)}{dp_E} < 0$, the household participation rate decreases with the

energy price. If $\beta < \frac{c_1}{q_R}$, $\frac{d(D_E)}{dp_R} > 0$, the household participation rate increases with the energy price

p_R ; whereas if $\beta > \frac{c_1}{q_R}$, $\frac{d(D_E)}{dp_R} < 0$, the household participation rate decreases with p_R . This result

reveals that when $\beta < \frac{c_1}{q_R}$, the household participation rate will increase with respect to the energy price

difference $p_R - p_E$. This suggests that if the effectiveness of reward points is big enough (i.e., $\beta > \frac{c_1}{q_E}$

), then the EDC does not need to reduce the energy price to attract more households to participate in the energy-conservation program. On the other hand, if the effectiveness of reward points is not big enough, then the attractiveness of the energy-conservation program itself will have more positive impact on the household participation rate.

In Table A1 note (8) and note (9), based on Eq. (15) and (29), we have:

$$\begin{aligned} \frac{d\pi_{LRP+EDC}}{dp_R} &= \frac{d\pi_{EDC}}{dp_R} & (A13) \\ &= q_R + \frac{K}{2} \left[\frac{d(H)}{dp_R} \right] + \frac{H}{2} \left[\frac{d(K)}{dp_R} \right] + \frac{k_1\beta H}{2c_1} \left[\frac{d(H)}{dp_R} \right] + \frac{c_1 K}{2k_1\beta} \left[\frac{d(K)}{dp_R} \right] \\ &= q_R \times \left[\frac{2c_1 - (c_1 K + k_1\beta H)}{2c_1} \right] + \frac{c_1 K + k_1\beta H}{2\beta} \end{aligned}$$

$$\begin{aligned} \frac{d\pi_{LRP+EDC}}{dp_E} &= \frac{d\pi_{EDC}}{dp_E} & (A14) \\ &= \frac{K}{2} \left[\frac{d(H)}{dp_E} \right] + \frac{H}{2} \left[\frac{d(K)}{dp_E} \right] + \frac{k_1\beta H}{2c_1} \left[\frac{d(H)}{dp_E} \right] + \frac{c_1 K}{2k_1\beta} \left[\frac{d(K)}{dp_E} \right] \\ &= q_E \times \left[\frac{c_1 K + k_1\beta H}{2c_1} \right] - \left[\frac{c_1 K + k_1\beta H}{2\beta} \right] \end{aligned}$$

(A13) and (A14) show that if $\beta > \frac{c_1}{q_E}, \frac{d(\pi_{LRP+EDC})}{dp_E} > 0$, the overall system profit increases with the

energy price p_E ; whereas if $\beta < \frac{c_1}{q_E}, \frac{d(\pi_{LRP+EDC})}{dp_E} < 0$, the overall system profit decreases with p_E .

If $\beta < \left(1 + \frac{2c_1}{c_1K + k_1\beta H - 2c_1}\right) \times \frac{c_1}{q_R}, \frac{d(\pi_{LRP+EDC})}{dp_R} > 0$, the overall system profit increases with the

energy price p_R ; whereas if $\beta > \left(1 + \frac{2c_1}{c_1K + k_1\beta H - 2c_1}\right) \times \frac{c_1}{q_R}, \frac{d(\pi_{LRP+EDC})}{dp_R} < 0$, the overall system

profit decreases with the energy price p_R . This result reveals that when

$\beta < \min\left[\left(1 + \frac{2c_1}{c_1K + k_1\beta H - 2c_1}\right) \times \frac{c_1}{q_R}, \frac{c_1}{q_E}\right]$, the overall system profit will increase with respect to

energy price difference $p_R - p_E$, which suggests that if the effectiveness of reward points is big enough

(i.e., $\beta > \max\left[\left(1 + \frac{2c_1}{c_1K + k_1\beta H - 2c_1}\right) \times \frac{c_1}{q_R}, \frac{c_1}{q_E}\right]$), there is no need to increase the price gap

between the regular energy program and the energy-conservation program since both the household participation rate and the overall system profit will decrease with respect to $p_R - p_E$.